Reflections on Mathematics Teaching and How to Improve It

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It is interesting that so few efforts to improve education have sought to directly improve the quality of teaching, for if any reform is to improve student learning, it will have to pass through the final common pathway that is the classroom. The task may be difficult, but if our efforts do not improve the quality of teaching and learning in classrooms, they are unlikely to translate into improved student learning.

I will base my remarks on more than a decade of research examining classroom teaching of mathematics in various countries around the globe. I will briefly discuss this research, and then present three truths about teaching that have emerged from it. Finally, I will discuss the implications these truths have for improving teaching.

Background: The TIMSS Video Studies

The TIMSS Video Studies were conceived in 1992, just as plans were being made for the huge cross-national comparison of mathematics and science achievement known as TIMSS (Third International Mathematics and Science Study). U.S. officials involved in planning the study knew that American students would probably do poorly on tests of achievement, and so they sought to collect contextual data that might help explain the poor performance. One area of great interest was teaching: Might cross-national achievement differences be related to differences in how mathematics is being taught in other countries?

Because teaching is not easily studied with questionnaires—especially cross-culturally—we hatched the idea of a video survey, combining the characteristics of survey research (generally conducted on large representative samples) with video. Up to that point, video had been used only for small-scale qualitative studies of teaching. What we proposed was videotaping national probability samples of eighth-grade mathematics teachers in three countries.
countries: Germany, Japan, and the United States. (We did a second TIMSS Video Study in 1999, in which we included seven countries.) The design was very simple: One hundred mathematics teachers were randomly sampled in each country. A videographer traveled around each country for a whole school year, videotaping one lesson from each teacher. The hard part was figuring out how to analyze all that video.

Some might wonder why we would want to collect videos of national random samples of eighth-grade mathematics lessons. We had two simple goals. First, we wanted to get a picture of what “average” teaching looks like in the United States. This is an odd concept to most people: Why not videotape “good” teachers? Couldn’t we learn more by studying videos of excellent teaching? Although that, too, would be an interesting study, we wanted to find out about average teaching for one simple reason: Most students, by definition, experience average teaching. If we want to improve the quality of education, we need to understand what the common experience looks like. Our second goal was to compare mathematics teaching in different countries, to see what we might be able to learn that would inform efforts to improve teaching in our own country.

Three Truths about Teaching

Rather than presenting detailed findings here, I want to focus on three truths about teaching that have emerged from the research. (For detailed information, see, for example, Stigler and Hiebert 1999; Hiebert et al. 2005.) I believe these truths to be critically important if we want to improve teaching.

Truth #1: Teaching Is a Cultural Activity

The first thing that hit us as we watched and analyzed these hundreds of hours of video was that teaching within countries appeared to vary very little, whereas teaching across countries differed markedly. We expected that teaching in Japan might be more homogeneous—after all, Japan is far less diverse than the United States ethnically, linguistically, and socioeconomically, and has a centralized Ministry of Education that controls much of what happens in the nation’s schools. But what really surprised us was the lack of variability in teaching methods within the United States. Whether the lessons were videotaped in Montana, New York City, New Orleans, or California, eighth-grade mathematics classrooms looked almost identical throughout the country.
American teachers often report that one thing they like about their jobs is the autonomy: when they close their classroom doors, they can teach however they deem best. But even given this apparent autonomy, it turns out that when they close their doors, they tend to do the same thing that every other teacher does. We believe that the reason for this is that teaching is primarily a cultural activity, meaning that it is learned implicitly through years of participation. They may be taking methods courses in their teacher education programs, but the primary influence on how teachers teach is the way they were taught as students. It doesn't take long for students to internalize the “script” that defines a mathematics lesson in their country, and once it's internalized, it seems as if it’s the natural way to teach.

We found that different scripts appeared to govern mathematics lessons in each country. In the United States, for example, a mathematics lesson typically follows this sequence: It starts by going over the homework from the day before, putting up and correcting any problems that students had trouble with. Then the teacher introduces the new skill to be taught, generally by working through an example problem on the board. Often the teacher will ask students to suggest what the next step might be. Rarely will the teacher explain how the procedure relates to important mathematics concepts. Once the whole class has gone through an example together, the teacher will have students try a few problems on their own, walking around to help any students who are having difficulty. Finally, if there is time left, the teacher may suggest that students get started on their homework for the next day.

A Japanese classroom looks very different. Lessons generally start with the teacher posing a difficult problem for students to work on, either alone or in groups. Significantly, the problem is one that students have not been taught how to solve. (In the United States, teachers rarely give students a problem to solve before they have taught them, step-by-step, how to solve that particular type of problem.) Students struggle with the problem and some appear confused—after all, they have not been taught the material before. Once students have come up with some solutions, the teacher calls several students to the board to present different solution methods. Sometimes the teacher will even send a student to the board to present an incorrect solution method, something that virtually never happens in U.S. classrooms. The teacher then leads a discussion of the different methods, comparing the methods to each other and explaining them in relation to previously learned mathematical concepts and ideas. Finally, the teacher may end with a brief lecture that summarizes and ties together what has been learned in the class.
Because cultural activities are learned implicitly, they are hard to identify as such; things that everyone around us is doing do not strike us as worthy of mention until we compare them with things that other cultures are doing. Cultural activities evolve over long periods of time and are multiply determined—that is, they are supported by curriculum, textbooks, student expectations, parent expectations, and so on. Because so much conspires to keep them in place, cultural activities are hard to change, which certainly corroborates the experience of anyone who has ever tried to change the way teachers teach.

Truth #2: There Are Many Ways to Teach Effectively

The second truth about teaching emerged from our second TIMSS Video Study. In the first study we only included three countries, and only one of the three (Japan) was high achieving based on the international achievement data. Thus, with only one model of teaching associated with high levels of student learning, we could not absolutely conclude that the methods used in that model were necessary for high achievement. To draw that conclusion would require that we study a variety of high achieving countries and look to see what they have in common in their teaching practices that could differentiate them from lower achieving countries such as the United States. We thus undertook a much larger study in which we compared the United States to six high-achieving countries: the Czech Republic, the Netherlands, Switzerland, Hong Kong, Australia, and Japan.

Did the other high-achieving countries use teaching methods similar to those used in Japan? In a word, no, they did not. What we found instead was great variation in teaching among the high-achieving countries for most of the variables we studied. In other words, there is not one way to teach effectively but many. Teachers in the Czech Republic and Hong Kong, for example, spend much of their time teaching the whole class, whether through lecture or recitation. Teachers in the Netherlands, on the other hand, have students work independently for much of the lesson, sometimes intervening very little in the flow of work. Many of these surface-level features of teaching, which reformers have spent so much time debating in our own country, appear unrelated to levels of student achievement cross-nationally.

It would be simpler if we had found a magic list of teaching features (sometimes known as “best practices”) that could provide us with a recipe for high achievement. But such is not the case. We must look more deeply to see what, if anything, the high-achieving countries share in terms of their teaching practices.
Truth #3: Teaching Quality Is Defined Not by What Teachers Do but by the Learning Opportunities They Create for Students

The longer and deeper we looked, the more we saw that despite differences on the surface, there were some commonalities among the high achievers. The key to finding these deeper similarities is to shift our focus from the teacher and what teachers do to the students and the kinds of learning opportunities that teachers create during classroom instruction. Although teachers across the high-achieving countries employ a variety of strategies and routines, the effect of these strategies and routines was similar across the high-achieving countries: somehow the teachers found a way to engage students in active struggle with core mathematics concepts.

Let us present one finding from the TIMSS 1999 Video Study to illustrate what we mean. One of the things we did in the new study was mark the beginning and end of each mathematics problem worked on during the lesson. Agreeing on what counts as a problem was not an easy task, but somehow we managed to come up with a definition that coders from seven different countries could agree on and use to code reliably. We found that across all countries, students in eighth-grade mathematics classes spend approximately 80 percent of their time working on problems, whether independently, as part of a small group, or as part of the whole class (Hiebert et al. 2005).

We categorized the problems into three types: stating concepts, using procedures, and making connections. Stating-concepts problems were the simplest of all, such as if the teacher asked the class, “What is the formula for the area of a triangle?” The “problem” in this case is simply to retrieve the formula $\frac{1}{2} b \times h$ from memory. Using-procedures problems were most common in all countries. In these problems, students practiced procedures that they had already been taught how to do by the teacher. For example, the teacher might hand out a worksheet with ten triangles, dimensions labeled, and ask students to use the formula to calculate the area of each triangle.

Making-connections problems are rich problems that seem to hold more potential for engaging students in deep thinking about mathematical concepts. For example, to stay with the same mathematical domain, a Japanese teacher was observed to start the first lesson on area of a triangle by handing out a large piece of chart paper to each student. On the chart paper was printed a number of line drawings of triangles of different types: acute, scalene, equilateral, right, and so on. Significantly, no dimensions were labeled: no numbers to indicate the length of sides, no dotted lines to indicate height. Students were told they could cut, draw, fold, or have a second sheet of triangles if they wished. Their task was to try to find a method for calculating the area of
the triangles, which they could then persuade the class would work for any triangle. Notice that they were not asked to calculate any of the areas—only to find a method that would work. Reasoning through this problem led students to consider the composition and decomposition of various geometric shapes, and led them to conjecture about a formula that would work no matter the type of triangle.

The two most common types of problems presented across all countries (using procedures and making connections) are presented in Figure 1. One thing is clearly evident in this graph: Japan is a real outlier in terms of the percent of problems categorized as making connections. Japan is the only country for which the majority of problems presented are of the making-connections type. All countries present some making-connections problems, but in every country except Japan, the most commonly presented problems are using procedures. This is an important finding because it shows that it is not necessary to present as many rich problems as is done in Japan to reach high levels of student achievement. Other high-achieving countries look more like the United States in terms of kinds of problems presented. The United States, in fact, is right in the middle on this dimension, and the two highest achievers in the group—Hong Kong and Japan—are at opposite extremes in terms of the percentage of making-connections problems they present to students.

To see what the high-achieving countries have in common, we must move beyond the surface—that is, the types of problems presented—and examine
in detail how teachers are using the problems to create learning opportunities for students. Just because a problem has the potential to engage students in core mathematical ideas does not mean that students will necessarily be so engaged. In fact, when we looked more closely at the making-connections problems and how they were implemented during instruction, we began to see a clearer picture of why the high-achieving countries might be so successful. Once a making-connections problem was presented, it could be skillfully implemented by the teacher and maintain its making-connections properties, or it could be changed into a different type of problem, most often a using-procedures problem. Take the example of the making-connections area-of-a-triangle problem described previously. A teacher concerned about the confusion that might emerge among students who find the problem too challenging might intervene soon after presenting the problem to provide a hint: “Try \( \frac{1}{2} b \times h \) if you get stuck.” If this were to occur, it would clearly change the problem from the students’ point of view from one that required deep thinking and struggle to one that required them only to fill in numbers in a formula and calculate the result.

In fact, this kind of transformation of the problem happened quite frequently in the United States but much less frequently in the high-achieving countries. Figure 2 shows the making-connections problems according to how they were implemented during the lesson—either as making-connections problems
or somehow transformed into using-procedures problems. Focus on the comparison of Hong Kong and Japan. Whereas in Figure 1 they looked starkly different, in Figure 2 they look almost identical. What this shows is that the two highest-achieving countries in our study differed markedly in the number of making-connections problems they presented but were quite similar in what they were able to accomplish in the classroom once they presented one of these rich problems. In both Hong Kong and Japan, nearly half of the making-connections problems were implemented as such, providing students with the opportunity to struggle with important mathematics concepts. The Netherlands and the Czech Republic showed a similar pattern. But the United States was a distinct outlier: none of the making-connections problems presented during the lessons we videotaped were presented as such, and well more than half were transformed during the lesson into using-procedures problems.

Thus, we can draw the following conclusion: Teaching is effective when learning opportunities are created for students, which in mathematics entails opportunities to struggle with important mathematics concepts. There are a variety of teaching strategies that can create such opportunities; which strategies are most effective will vary depending on the context.

**Improving Teaching**

These three truths have implications for how to go about the task of improving teaching. I will briefly discuss three of these implications.

**Implication #1: Shift Focus from Teachers to Teaching**

There is currently great policy emphasis on improving teaching. However, the most commonly suggested means of improving teaching is to recruit different—that is, smarter and better prepared—people into the profession. If teaching is cultural, then it is unlikely that different people will teach in fundamentally different ways than do our current teachers. Thus, we need to shift our focus from who the teachers are to what the teachers are actually doing in the classroom. Improvements in teaching will come when we are able to make incremental and sustainable improvements in the cultural routines that define students’ learning opportunities in the classroom.
Implication #2: Define “Best Practices” Not by What Teachers Do but by the Learning Opportunities They Create for Students

Even as we shift our focus from teachers to teaching, we must rethink what we mean by quality teaching. Our research shows that quality is defined not by what teachers do but by their ability to create learning opportunities for students. So we must move away from prescribing certain “best practices” and toward understanding how different kinds of practices affect learning in different situations.

Implication #3: Define Teaching as a Learning Profession

Finally, if quality teaching means first selecting the right teaching strategy for the right situation at the right time, then being able to implement that strategy successfully to improve student learning, then the job of teaching is far more challenging than many would hope. Teaching is not about following best practices but about having a large repertoire of strategies, as well as the judgment and analytic skills to be able to fit the strategies to the context. If this is the job of teaching, then we must redefine teaching to be a learning profession, and we must provide teachers with opportunities for continuous learning. This means creating stable settings (time, place, and colleagues) within the teacher’s workweek that can be devoted to working with colleagues to improve teaching. If teachers are not given opportunities to learn, we should not expect them to be able to create ever more powerful opportunities for students to learn.

References
